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The orientation kinetics of real magnetic particle assemblies

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Orientation of the magnetic particle assemblies in the external magnetic field is considered. Time dependence of the average magnetization and orientation ratio are calculated. Interaction effects and the influence of the particle size distribution are discussed.

1. Introduction

Orientation of magnetic particle assemblies is now the important part of the magnetic storage technology based on the particulate media. The problem was previously studied in refs. [1–5], but some important questions are still open. The purpose of this paper is to complete the theoretical study of the magnetic particle orientation, including interparticle interaction and particle size distribution effects.

2. Non-interacting (ideal) particle ensemble

Some aspects of the behavior of ideal ensembles were already studied in refs. [1–4], but to make our paper self-consistent and to introduce the notation we will write down the basic equations.

The equation of motion for the ellipsoidal magnetic particle with volume V and first uniaxial anisotropy constant K can be written as

$$f_R\dot{\theta} = -KV\sin(2\psi), \tag{1}$$

where θ is the angle between the anisotropy axis $\bar{n}$ and the external field, ψ the angle between the same axis and the particle moment and f_R denotes the rotational friction coefficient of the particle. Further we will use dimentionless anisotropy constant $\beta = 2K/I_s^2$ and the dimensionless field $h = H/\beta I_s$, where I_s denotes the saturation magnetization.

Angle ψ can be found from the equilibrium equation for the magnetic moment

$$h_z\sin(\theta - \psi) = 0.5\sin(2\psi) \tag{2}$$

and therefore we have to solve the nonlinear ordinary differential equation

$$f_R\dot{\theta} = -0.5\beta I_s^2 V\sin(2\psi(\theta)). \tag{3}$$

For arbitrary external field, eqs. (2) and (3) can be solved only numerically. In ref. [3] results of such a numerical solution are shown only for a sufficiently large external field $h_0 > 1$, or $H_0 > 2K/I_s$.

To complete the consideration of the ideal ensemble, let us consider other cases. First of all, for weak ($h_0 \ll 1$) and strong ($h_0 \gg 1$) external fields eqs. (2) and (3) can be solved analytically.

Weak external field. For ($h_0 \ll 1$) the deviation ψ of the magnetic moment from the anisotropy axis is small ($\psi \ll 1$) and eq. (3) can be rewritten as

$$f_R\dot{\theta} = -h\beta I_s^2 V\sin\theta. \tag{4}$$

The solution of this equation is $\mathrm{tg}(\theta/2) = \mathrm{tg}(\theta_0/2)\exp(-\gamma t)$ where θ_0 corresponds to the initial particle orientation. The relaxation rate $\gamma = h\beta I_s^2 V/f_R = HI_s^2 V/f_R$ does not depend on the magnetic anisotropy constant, for the reason that in the large anisotropy limit magnetic moment is always aligned almost in the easy-axis direction, independent of the concrete value of the anisotropy.

For the applications two quantities are of greatest interest: the reduced magnetization $m_z = M_Z/I_s$ and the orientation ratio $r = j_{R\parallel}/j_{R\perp}$ (see ref. [3] for details). For the initally uniform distribution of particle easy-axes we can easily obtain

$$m_z(t) = \mathrm{cth}(\gamma t)[1 - (2\gamma t)/\mathrm{sh}(2\gamma t)], \tag{5}$$

$$r(t) = j_z/j_x, \tag{6}$$

$$j_z = \mathrm{cth}(\gamma t)\left[1 + \frac{2}{sh(2\gamma t)}\ln\left((1 + \mathrm{e}^{-2\gamma t})/2\right)\right],$$

$$j_x = \frac{2\mathrm{e}^{-2\gamma t}}{(1 + \mathrm{e}^{-2\gamma t})^2}.$$

Strong external field. The opposite condition ($h_0 \gg 1$) leads to the equation of motion

$$f_R\dot{\theta} = -0.5\beta I_s^2 V\sin(2\theta), \tag{7}$$

with the solution $\mathrm{tg}\theta = \mathrm{tg}\theta_0\exp(-\gamma t)$, where the relaxation rate $\gamma = \beta I_s^2 V/f_R$ does now not depend on the

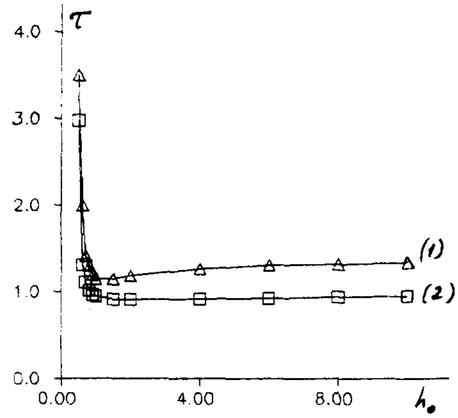

Fig. 1. Magnetization (1) and orientation (2) time dependences vs external field.

external field for the same reason, as described above. The average magnetization and orientation ratio are

$$m_z(t) = (1 - e^{-\gamma t})^{-1},$$

$$r(t) = j_z/j_x,$$

$$j_z = (1 + e^{-\gamma t})^{-1},$$

$$j_x = (2/\pi)\int_0^1 \sqrt{\frac{1 - x^2}{1 + x^2(e^{2\gamma t} - 1)}}\,dx.$$

Arbitrary external field. To describe the behavior of the system for the arbitrary field, let us introduce
i) the magnetization time $\tau_{0.8}^m$, defined as the time required to achieve the magnetization $m_z = 0.8$; and
ii) the orientation time τ_2^r, defined as the time required to obtain the orientation ratio $r = 2$.

The dependences $\tau_{0.8}^m$ and τ_2^r versus the external field are shown in fig. 1. The most interesting feature of the system is the nonmonotonical behavior of this curves: there is the optimal field value for the orientation process, i.e. the dependence $\tau_{0.8}^m(h_z)$ has a minimum in the vicinity of $h_z \approx 1.0$. This fact was already described in ref. [3], but without any physical explanation.

This nontrivial behavior (magnetization time increases with the field!) can be explained using the fact, that the force moment N, acting on the particle, depends not directly on the magnetic field value, but on the angle ψ between the particle easy-axis and the magnetic moment. This dependence for the uniaxial anisotropy is not monotonic ($N_{an} = KV \sin(2\psi)$): $N_{an}(\psi)$ has a maximum for $\psi = \pi/4$ and decreases to zero for $\psi \to 0$ and $\psi \to \pi/2$. For this reason, when for a strong external field the magnetic moment deviates from the easy-axis so that $\psi > \pi/4$, the anisotropy force moment and the orientation rate decrease. For the initially uniform spatial distribution of easy axes at $t = 0$ for the most of particles $\theta = (\bar{n}, \bar{H}_0) > \pi/4$ and therefore for strong applied field $\psi > \pi/4$ at the initial orientation stage. This leads to the decrease of the force moment and to the increase of the magnetization time τ.

3. Interparticle interaction

Interparticle interaction effects were studied in ref. [4], but the consideration in ref. [4] is incorrect even in the frame of mean-field approximation used there. To take the magnetostatic interaction between particles into account using the mean-field theory, we have to evaluate the interaction (local) field distribution density $\rho(\bar{h}_{loc})$, and then we can write any physical quantity of interest as

$$f(\bar{h}_0) = \int f^{(0)}(\bar{h})\rho(\bar{h} - \bar{h}_0)\,d\bar{h}, \tag{8}$$

where $f^{(0)}(\bar{h})$ is the corresponding dependence for the ideal ensemble, i.e., without interparticle interaction.

This mean-field approximation is valid (see ref. [6]) for the low volume concentration $\eta \ll 1$ of magnetic particles and for the large anisotropy constant $\beta \gg 1$. In our case we usually have $\eta \sim 0.1$–0.4 and the anisotropy constant $\beta \approx 2\pi$ for long ellipsoids. Therefore we can expect, that the mean-field approximation provides a reasonable semiquantitative estimation of the interaction effects.

To obtain qualitative results, we can use the 1d-version of eq. (8), namely:

$$f(h_{0z}) = \int f^{(0)}(h_z)\rho(h_z - h_{0z})\,dh_z. \tag{9}$$

It was shown [6], that $\rho(\bar{h}_{loc})$ can be approximated sufficiently good with the restricted Lorentzian distribution with the restriction field $h_{max} \sim 1/\beta \ll 1$. For this reason for smooth functions $f(h)$ we can use the Taylor expansion near $h = h_0$. Substituting this expansion into eq. (9) we obtain

$$f(h_{0z}) = f^{(0)}(h_{0z}) + \frac{\sigma_z^2}{2} f''^{(0)}(h_{0z}) + \cdots, \tag{10}$$

where $\sigma_z^2 = 2\pi^2\eta/15\beta^2$ is the dispersion of the z-component of the local field [6]. For the anisotropy constant $\beta \approx 2\pi$ we obtain $\sigma^2 \approx \eta/30$, which leads to the conclusion, that for any reasonable volume concentration interaction effects can be neglected. This means, that the strong dependence of the orientation time on the particle concentration cannot be explained only with the interparticle interaction, but probably is due to the aggregation phenomena and other structural changes in the magnetic particle assemblies.

4. Particle size distribution effects

We consider the ensemble of particles of the same diameter and with the log-normal length distribution

$$\rho(x) = \frac{1}{\sqrt{2\pi}\sigma_x x} \exp\left(-\frac{\ln^2(x/x_0)}{2\sigma_x^2} \right).$$

Then for any size-dependent quantity $f(x)$ we can write an expression similar to eq. (10) and with the Taylor expansion near $x = x_0$ we obtain

$$f(\bar{x}) = f^{(0)}(\bar{x}) + \frac{\sigma_x^2 x_0^2}{2} f''^{(0)}(x_0) + \cdots, \tag{11}$$

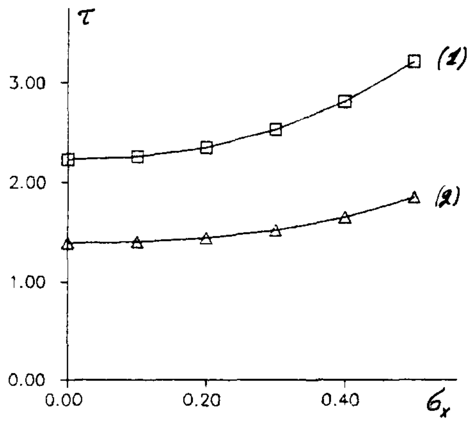

Fig. 2. Magnetization (1) and orientation (2) time vs distribution width for ensemble with the particle size distribution.

where $\bar{x} = x_0 \exp(\sigma_x^2/2)$ is the average particle length. In contrast with eq. (10), there are no reasons to consider σ_x as small parameter: real assemblies have usually sufficiently wide size distribution, so that $\sigma \sim 0.1$–0.4.

The evaluation of the magnetization time dependence for such an ensemble (with log-normal length distribution) in the strong field limit leads to the result

$$M_z(t) = \frac{1}{\sqrt{2\pi}\bar{x}\sigma_x} \int_0^\infty \frac{\xi \exp\left[-\ln^2(\xi/\xi_0)/(2\sigma_x^2) \right] \mathrm{d}\xi}{1 + \exp(-\bar{t}/\xi^2)},$$

where we have used dimensionless length $\xi = x/x_0$ and the dimensionless time $\bar{t} = t/\tau_0$ with $\tau_0 = (C\eta_0 x_0^2)/(2\pi I_s^2 S)$ (S is the area of the cross section perpendicular to the long axis, η_0 denotes the viscosity coefficient, $C \approx 0.5$ for long ellipsoids). The dependences for τ versus the distribution width are shown on fig. 2. In agreement with eq. (11) they are nearly quadratic.

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